A Qualitative Temporal Extension of Here-and-There Logic

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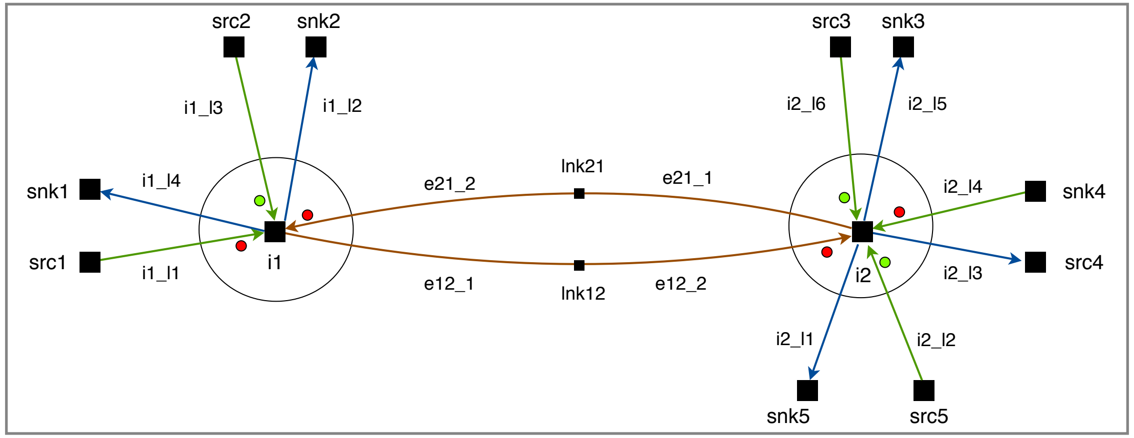
Motivation

- Traffic Congestions
  - Large problem in big cities worldwide
  - Estimate by WEF, cost for US economy: $87 billion (2018) [Fleming, 2019]
- Microscopic traffic simulations:
  - Used to simulate traffic, e.g., SUMO
- Mesoscopic Traffic Flow Model (MTF):
  - Simulation of traffic flow over networks
  - Static/dynamic component
  - Abstraction of time, road network, vehicles
  - Used for TL signal plan configuration encoded as ASP rules [Eiter et al., 2020]
- Develop diagnosis methods on top MTF:
  - Traffic diagnosis to find "faulty" patterns such as inefficient signal plans or accidents

Traffic in Singapore [The Strait Times, 2016]



MTF of two intersections [Eiter et al., 2020]

Motivation (cont.)

- Horizont: Model-based diagnosis in a temporal setting applied to traffic diagnosis
- Starting point: Temporal Behavioral Models [Console1997]:
  - Temporal behavior formulae and integrity constraints that describe causal relation between *explanations and observations under temporal constraints, i.e., using Allen's Interval Algebra*
  - Example:

$$water\_ret(T_1), therapy(absent) \ \ \textsf{explains}\ blood\_vol(high, T_2)\ \{T_1(before)T_2\}$$

- The original syntax is not well suited for complex causal relations:
  - Temporal relations could be combined with atomic formulas
  - Nested temporal formulas and negation desired
  - "explains" could be replaced with $\leftarrow$, $\rightarrow$, or $\wedge$ depending on the reasoning task, .e.g., deduction, abduction, or consistency checking

Motivating Example

$$(a \vee b) \wedge ((a \wedge \sim c)\ overlaps\ (o_1 \vee o_2)) \vee ((b \wedge c)\ overlaps\ (o_1 \vee o_2)) \wedge o_1 \wedge o_2$$

Two observations o_1, o_2 of low traffic caused by
either a (normal) congestion a or by an accident b,
while either roadworks c occurs or does not occur, denoted by $\sim c$
on overlapping time intervals

Preliminaries

R	Definition with start/end points
$x(p)y$	$before(x,y)=\{(x,y) : \underline{x} < \overline{x} < \underline{y} < \overline{y}\}$
$x(m)y$	$meets(x,y)=\{(x,y) : \underline{x} < \overline{x} = \underline{y} < \overline{y}\}$
$x(o)y$	$overlaps(x,y)=\{(x,y) : \underline{x} < \underline{y} < \overline{x} < \overline{y}\}$
$x(s)y$	$starts(x,y)=\{(x,y) : \underline{x} = \underline{y} < \overline{x} < \overline{y}\}$
$x(f)y$	$finishes(x,y)=\{(x,y) : \underline{y} < \underline{x} < \overline{x} = \overline{y}\}$
$x(d)y$	$during(x,y)=\{(x,y) : \underline{y} < \underline{x} < \overline{x} < \overline{y}\}$
$x(e)y$	$equal(x,y)=\{(x,y) : \underline{y} = \underline{x} < \overline{x} = \overline{y}\}$

- Allen's Interval Algebra (IA) [Allen1983]
  - Algebra/calculus to express temporal constraints
  - Domain is set of intervals defined over the linear order of $\mathbb{T}$
  - 13 basic relations hold between intervals
- Syntax: First-Order Here-and-There (FOHT) Logic [Heyting1930]
  - FO language: $\Sigma = \langle C, F, P \rangle$, constants/functions/predicates, (ground) terms, atoms, and literals are defined as usual
  - Two negations: $\sim$ and $\neg$ (strong and weak negation)
- Semantics: FOHT Logic
  - FOHT-model $M = \langle D_h, H, D_t, T \rangle$, where $C \subseteq D_h \subseteq D_t$ and $H \subseteq T \subseteq Lit$ such that there are no conflicting literals in T
  - Simpler with constant domain assumption $D_t = D_h$, FOHTc-model: $M = \langle D, H, T \rangle$
  - Satisfaction relation: $M, w \vDash \phi$ for $w \in \{h, t\}$, where we have a totally ordering of worlds, i.e., $h \leq h, h \leq t, t \leq t$

Preliminaries (cont.)

- Semantics: FOHT Logic (cont.)
  - Satisfaction relation for a sentence ϕ depends on its structure, e.g.:
    $\alpha \wedge \beta : M, w \models \alpha$ and $M, w \models \beta$
    $\alpha \rightarrow \beta$: for every $w' \geq w$: either $M, w' \not\models \alpha$ or $M, w' \models \beta$
  - A closed formula ϕ has a model M if $M, h \models \phi$ and $M, t \models \phi$
- Equilibrium Models [Pearce2004]
  - Introduce minimal models to FOHT; $M = \langle D, H, T \rangle$ is an Equilibrium Model of Π if: (1) M is a total model, i.e., $H = T$ (2) Π has no other model $\langle D, H', T \rangle$ with $H' \subset H$
- Quantified many-valued logic QN3 and QN5
  - Characterises model semantics using three/five-valued matrix of $\mathcal{T}_3 = \{-2,0,2\}$ and $\mathcal{T}_5 = \{-2,-1,0,1,2\}$
  - Interpretation function f_F for connectives F:

F	$x \wedge y$	$x \vee y$	$\sim x$	$\neg x$
f_F	$\min(x,y)$	$\max(x,y)$	$-1 \cdot x$	$\begin{cases} 2 & \text{if } x \leq 0 \\ -1 \cdot x & \text{otherwise} \end{cases}$

- Bijection between FOHT-models and QN5-valuations:
  for an atom $p \in H$, resp., $\sim p \in H$ the valuation $\sigma(p)$ is 2, resp., -2

Temporal Extension of HT Logic

- Extends Σ to $\Sigma^t = \langle C, F, P, \mathbf{A} \rangle$, where $\mathbf{A} \subseteq Lit \times (\mathbb{Z} \times \mathbb{Z})$ (time interval associations)
- Temporal assignment $\tau_A : Lit \to (\mathbb{Z} \times \mathbb{Z}) \cup \{\mathbf{u}\}$, where is $\mathbf{u}$ undefined time instance
- Allow nested formulas, need interval coalescing, e.g., $(a \wedge \sim b)\ overlaps\ (o_1 \vee o_2)$
- Two coalescing operators for $x = \tau_A(\alpha), y = \tau_A(\beta)$:

  $\tau^*_A(\phi)$ is the recursive application of it

x, y *satisfy*	$x(p)y$	$x(m)y$	$x(o)y$
$coal_\wedge(x, y)$	$\mathbf{u}$	$[\overline{x}, \overline{x}]$	$[\underline{y}, \overline{x}]$
$coal_\vee(x, y)$	$\mathbf{u}$	$[\underline{x}, \overline{y}]$	$[\underline{x}, \overline{y}]$

- Semantics: Characterised by QN3/QN5, define valuation for formula $\alpha\ \nu\ \beta$:

$$\sigma(\alpha\ \nu\ \beta) = \frac{1}{2} \cdot eval_\nu(\tau^*_A(\alpha), \tau^*_A(\beta)) \cdot \min(\sigma(\alpha), \sigma(\beta)),$$

where
(a) $eval_\nu(x, y) = 2$ if $IA_\nu(x, y)$ holds
(b) $eval_\nu(x, y) = -2$ if $IA_\nu(x, y)$ not holds
(c) $eval_\nu(x, y) = 0$ if $x = \mathbf{u} \vee y = \mathbf{u}$

- Example: The formula $(x \vee y) \wedge ((x\ before\ z) \vee (y\ before\ z))$ with a temporal assignment $\{(x, [1,2]), y, [2,3]), (z, [4,5])\}$, has the following equilibrium models:

  $i_1 : (\varnothing, \{(x, [1,2])\})$, $i_2 : (\{(x, [1,2]), (z, [4,5])\}, \{(x, [1,2]), (z, [4,5])\})$,

  $i_3 : (\{(x, [1,2]), (y, [2,3]), (z, [4,5])\}, \{(x, [1,2]), (y, [2,3]), (z, [4,5])\})$

Temporal Tableau Calculus

- Calculus for three-valued logic N_3, can be extended to quantified and N_5 logic [Pearce2000]
- Total models calculated by applying tableau rules with labels that are set of sets over $\mathcal{T}_3 = \{-2, -0, 2\}$
- Non-temporal tableau $\mathfrak{T}$ for a theory $\Pi = \{\phi_1, \ldots, \phi_n\}$ starts with initial tableau $\{2\}{:}\phi_1 \ldots \{2\}{:}\phi_n$
- Expansion rules with labels $S := \{\{2\}, \{0,2\}, \{-2,0\}, \{-2\}\}$, $S^- := \{\{-2,0\}, \{-2\}\}$, and S^+:

$$\frac{S^- : \phi \wedge \psi}{S^- : \phi \mid S^- : \psi} \qquad \frac{S^+ : \phi \wedge \psi}{\begin{array}{c} S^+ : \phi \\ S^+ : \psi \end{array}}$$

- Temporal tableau system for Π with initial tableau and temp. assignments $\tau_A(\phi)$:
- Temporal expansion rules depending on $E := eval_\nu(\tau^*_{A'}(\phi), \tau^*_{A'}(\psi))$:

$$\begin{array}{|l} \{2\} : (\phi_1)_{t_{\mathcal{A}}(\phi_1)} \\ \ldots \\ \{2\} : (\phi_n)_{t_{\mathcal{A}}(\phi_n)} \end{array}$$

$$\frac{S : (\phi \, \nu \, \psi)_{\mathcal{A}'}}{\begin{array}{c} S^+ : (\phi)_{\tau^*_{\mathcal{A}'}(\phi)} \\ S^+ : (\psi)_{\tau^*_{\mathcal{A}'}(\psi)} \end{array}} \qquad \tfrac{1}{2} \cdot E \cdot S = S^+$$

- A branch B of $\mathfrak{T}$ is closed if for any formula ϕ, there are labels such $S_1 : \phi$, $S_2 : \phi$ with $S_1 \cap S_2 = \emptyset$
- Satisfiability (SAT): A branch B of $\mathfrak{T}$ is SAT if for every formula $\phi \in B$ there is $\sigma(\phi_i) \in S_1 \cap \cdots \cap S_n$, and $\mathfrak{T}$ is SAT if at least one branch of $\mathfrak{T}$ is SAT.

Sound- and Completeness of the Calculus

- Soundness of a tableau system: $\vdash_{N_3} (\Pi, A)$ implies $\models_{N_3} (\Pi, A)$
  - Establishing that SAT is a loop invariant of the tableau system [Fitting1996]
  - We assume that $\mathfrak{T}$ is SAT and show that after applying an expansion rules, still SAT → case distinctions
- Completeness of a tableau system: $\models_{N_3} (\Pi, A)$ implies $\vdash_{N_3} (\Pi, A)$
  - Make use of generic machinery of “tableau for many-value logics“ [Hähnle1999]
  - Given an arbitrary $\phi = S : \gamma(\phi_1, \ldots, \phi_m)$, its (*sets-as-signs) disjunctive normal form (DNF)* representation is defined as $\psi = \vee_{i=1}^{l} C_i$ with $C_i = \wedge_{j=1}^{n_i} S_{i,j} : \psi_{i,j}$
  - Many-valued (mvs) Hintikka set Ω is a set of signed formulas such that: (1) it has no *closing* formula; (2) if $\phi \in \Omega$ and ψ is a *DNF* rep. of ϕ, then form some C_i it holds that $\{S_{i,1} : \psi_{i,1}, \ldots, S_{i,n_i} : \psi_{i,n_i}\} \subseteq \Omega$
  - Every mvs-Hintikka set Ω has a model [Hähnle1999]
  - Our tableau rules have already the generic schema of Hähnle, where $F_i = C_i$:

$$\frac{S : \gamma(\phi_1, \ldots, \phi_m)}{F_1 \mid \cdots \mid F_l .}$$

- Temporal extension needed to be adapted to encoding of DNF representation:
  A temporal formula $(\alpha \; \nu \; \beta)$ can be viewed as a formula $S : \gamma_E(\alpha, \beta)$ depending on E and A

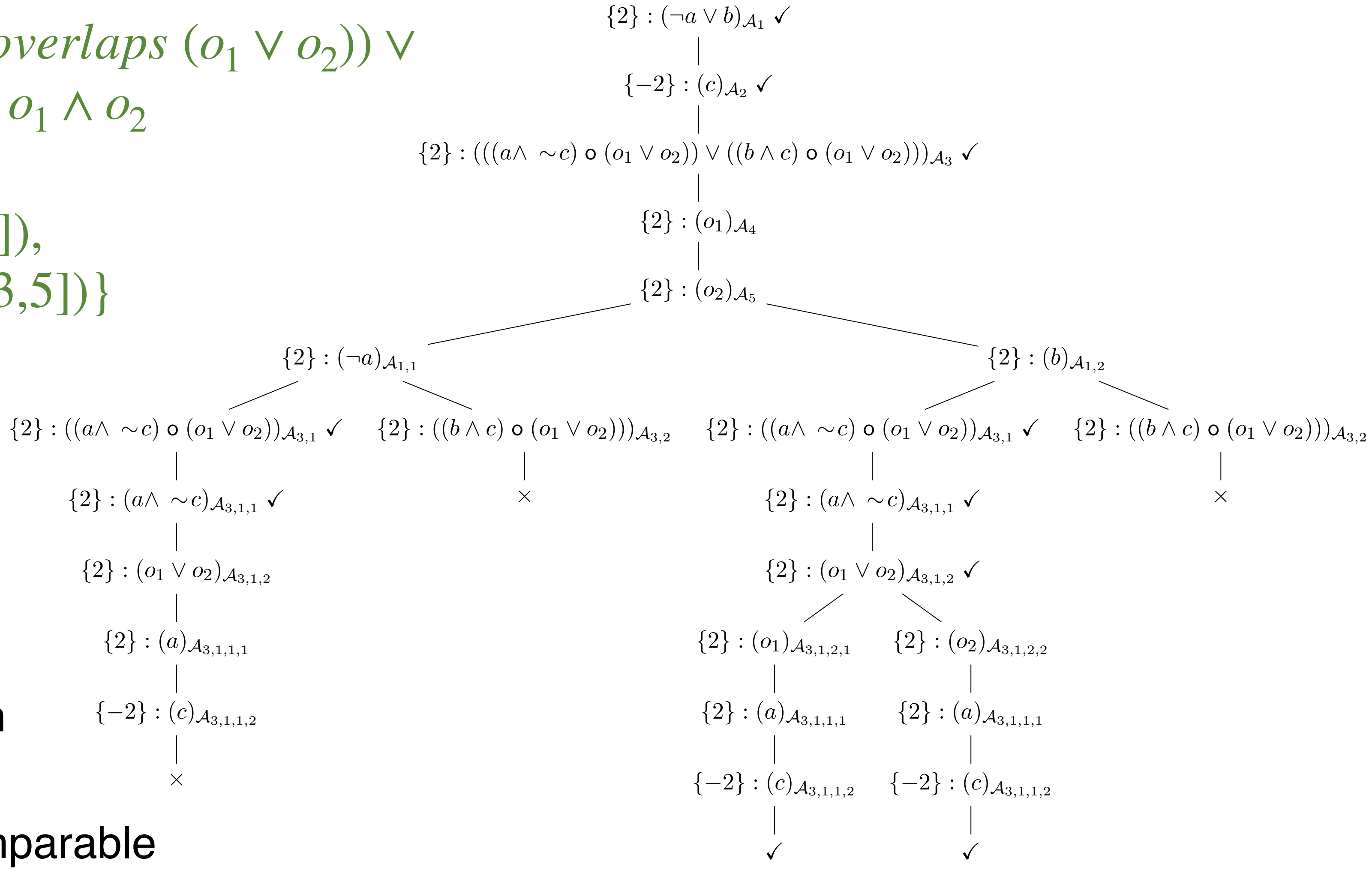
Conclusion/Discussion

- Prototypical Implementation to show feasibility:
  - Implemented in Python 3.7 with temporal CNF/DNF as input
  - Optimisation techniques not applied yet
  - Available on: https://github.com/patrik999/ EL-TempTableau
- Conclusion:
  - A new semantics, tableau calculus, and related solver for Temporal Behavioral Models
  - Arbitrary nesting of formulas, coalescing and undefined time instances
  - Initial steps towards an abduction-based temporal diagnosis framework
  - See our LPNRM'22 paper for more details
- Future/Ongoing work:
  - Add minimality checking to tableau system → use of sub-tableau rules [Pearce2000]
  - Richer nesting in temporal formulas, several intervals in temporal assignments
  - Improved implementation + using optimization techniques
  - Encode directly in ASP and use one of the existing solvers (unravelling nesting)

THANKS FOR YOUR ATTENTION!

Case Study

- Case study of introductory example
  - $(\neg a \vee b) \wedge \sim c \wedge ((a \wedge \sim c)\ overlaps\ (o_1 \vee o_2)) \vee ((b \wedge c) overlaps\ (o_1 \vee o_2)) \wedge o_1 \wedge o_2$
  - $A = \{(a, [1{,}4]), (b, \mathbf{u}), (c, [1{,}2]), (\sim c, [3{,}5]), (o_1, [1{,}3]), (o_2, [3{,}5])\}$



- Evaluation:
  - Two models derived from open branches
  - Runtime, below 1s but not comparable to sota tableau solvers

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